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CHARACTERISTICS OF INERTIALLY STRETCHING SHAPED-CHARGE JETS IN FREE FLIGHT

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Metal shaped-charge jets (SCJ) that are high-speed elongating axisymmetric bodies are known to be formed as a result of explosion of a shaped charge lined with a thin layer of metal [1, 2]. The process of explosive formation is characterized by the difference in axial velocity across such jets. The velocity of the head elements is of the order of the orbital velocity, and the tail elements move, as a rule, at a velocity of about 2 km/sec. The axial-velocity distribution along SCJ gives the initial gradient $\dot{\varepsilon}_{z0}$ (initial axial strain rate) the local value of which is determined by the ratio of the axial-velocity difference ΔV_z to the initial length $\Delta l_{\rm el}$ of an SCJ element. This length is usually assumed to be equal to the length of the corresponding section of the forming metal lining (shaped-charge lining). The initial axial-velocity gradient varies along the SCJ and determines the initial strain rate of the jet elements. For most SCJ, the characteristic values of the initial gradients are 10^4–10^5 sec^{-1}.

Under the action of the velocity gradient, SCJ in free flight are stretched in the axial direction with simultaneous decrease in their longitudinal sizes. At the initial stage of their existence, most jets are characterized by homogeneous stretching throughout the length without concentrated deformation and with retention of a near-cylindrical or slightly conical shape. Figure 1 shows an x-ray pattern of an SCJ generated by a shaped charge at 70 μsec from the beginning of an explosion under laboratory conditions. In some cases, this stage is called the inertial stage. Stretching is then gradually localized in regions with many necks formed in the jet (the neck stage of stretching). As a result, the SCJ breaks up into a certain number of separate elements which do not vary in length thereafter. Figure 1b shows an x-ray pattern of the same SCJ as in Fig. 1a, but for the later time.

The character of SCJ breakdown into separate elements is different and depends on the characteristics of the material of the shaped-charge lining and on the geometrical and kinematic characteristics of SCJ such as the initial radii of the elements and the initial axial-velocity gradient. For example, high-gradient copper jets break up into elements in a surprisingly regular fashion, which is similar to the x-ray pattern of Fig. 1b. The formation of geometrically similar separate elements with a developed neck with nearly zero radius is characteristic of this type of destruction (plastic failure). Failure of SCJ made of nickel, niobium, and pure aluminum occurs similarly.

Failure of jets made of other materials is of a different character. For example, SCJ made of lead or tungsten fail in volume. This is illustrated by a lead jet in the x-ray patterns of Fig. 2 at three successive moments of time. In most cases, failure of jets that are formed by steel-lined shaped charges occurs as a "quasi-brittle" breakup, that is, without the formation of a pronounced neck. In some cases, a similar kind of failure is also typical of copper jets. Figure 3 shows an x-ray pattern of a massive low-gradient copper SCJ at the moment of its breakup.

The so-called coefficient of ultimate elongation, which is determined by the ratio of the total length of a jet element after breakup to its initial length, $n_{\rm ult} = \Delta l_{\rm ult}/\Delta l_{\rm el}$, is a quantitative estimate of the ability of a SCJ to elongate without breakup. A common property of all materials under conditions of SCJ is their anomalously high plasticity in comparison with the static conditions. For example, after breakup the total length of the elements of a high-gradient copper SCJ is approximately a factor of 10 larger than the initial

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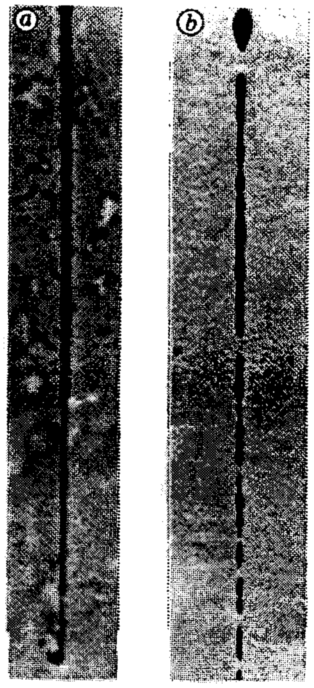

Fig. 1

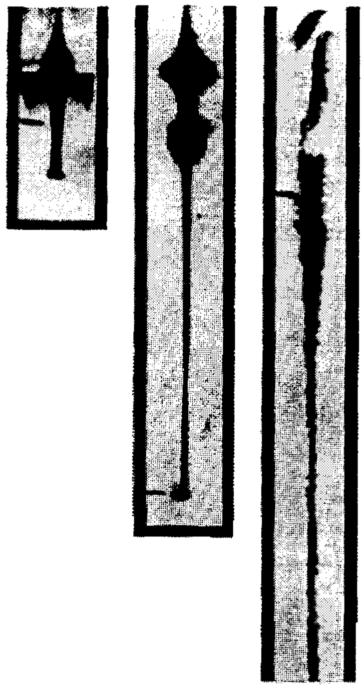

Fig. 2

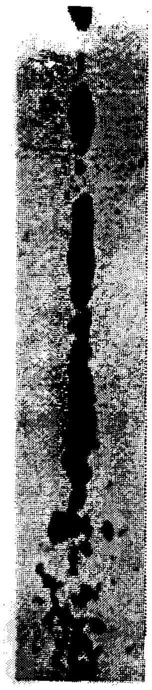

Fig. 3

length of the SCJ. Some sections of the SCJ undergo even a more considerable elongation prior to the breakup. For example, the value of the coefficient of ultimate elongation for niobium can reach $n_{\text{ult}} = 26$. The so-called coefficient of inertial elongation n_{i}, which separates the inertial and neck stages of SCJ deformation, is regarded sometimes as an intermediate characteristic of the process. The number N and the sizes of separate elements formed during failure are also the basic quantitative characteristics of SCJ breakup.

Stretching of a SCJ in free flight precedes its interaction with a barrier and determines directly the through-piercing action of the jet [2]. In fact, the known high penetrative power of shaped charges is realized precisely at the stage of jet stretching, and this apparently explains the great interest in this question in the literature (see, for example, [3–9]).

In the present paper, we consider the behavior of a SCJ at the initial stage of its deformation when the jet elements retain a near-cylindrical shape, and the process of deformation localization and neck development has not yet started. The main results were obtained from computations on the basis of continuum mechanics. Not giving an answer to the question on the character of and on the reasons for a fairly definite breakup of the SCJ into separate elements, the results obtained, nevertheless, make it possible to determine the character of evolution of the stress-strain state of the SCJ during its inertial deformation and to get a preliminary insight into the mechanism of this process. The latter is of importance for clarifying the behavior of the SCJ in free flight and especially because it is impossible to measure directly the stress-state characteristics in superhigh-speed explosion-formed bodies.

Let us consider the stretching of a cylindrical bar with initial length l_0 and radius R_0 under the action of the axial-velocity gradient V_z with its initial linear distribution $V_z = \dot{\varepsilon}_{z0} z$. This distribution is characterized by the initial strain rate $\dot{\varepsilon}_{z0} = V_0/l_0$, where V_0 is the difference in axial velocities between the ends of the element and z is the axial coordinate reckoned from one of these ends. A similar model corresponds to an arbitrarily isolated SCJ element at the stage of uniform stretching which is investigated in the frame of reference related to one of its ends. This end is assumed to be fixed in the axial direction.

An analytical solution of the problem of determining the stress-strain characteristics of such a bar is obtained under the following assumptions on the conditions of deformation and material characteristics: the bar material is incompressible ($\rho = \rho_0 = \text{const}$), inelastic-perfectly-plastic with yield point $Y_0 = \text{const}$, and is subject to the von Mises yield condition; stretching of the bar occurs at a constant value of the Lagrangian axial-velocity gradient $\partial V_z/\partial z_0 = \text{const}$, where z_0 is the axial Lagrangian coordinate of the bar particles. The latter assumption corresponds to the condition that the jet element is uniformly stretched with a constant difference in the velocity $V_0 = \dot{\varepsilon}_{z0} l_0$ between the plane cross sections that bound the element in question or at a constant absolute axial velocity, which is in agreement with experimental data. A natural consequence of this assumption is retention of the cylindrical shape of the bar during its stretching.

With allowance for the assumptions made above, one can establish an obvious relationship between the current and initial dimensions of the bar: $lR^2 = l_0 R_0^2$. Differentiating this relation with respect to time and extending the result to an arbitrary particle of the bar with the Eulerian radial coordinate r, we obtain an expression for the radial velocity component: $V_r = -0.5r\dot{l}/l$. When the coefficient of current elongation of the bar is defined as

$$n = l/l_0 = (l_0 + V_0 t)/l_0 = 1 + \dot{\varepsilon}_{z0} t, \tag{1}$$

the expressions for the radial- and axial-velocity components take the form

$$V_r = 0.5r\dot{\varepsilon}_{z0}/(1 + \dot{\varepsilon}_{z0} t), \qquad V_z = V_0 z/l = \dot{\varepsilon}_{z0} z/(1 + \dot{\varepsilon}_{z0} t). \tag{2}$$

The components of the strain-rate tensor are determined in accordance with the known kinematic relations:

$$\begin{gathered} \dot{\varepsilon}_r = \partial V_r/\partial r = -0.5\dot{\varepsilon}_{z0}/(1 + \dot{\varepsilon}_{z0} t), \qquad \dot{\varepsilon}_\theta = V_r/r = -0.5\dot{\varepsilon}_{z0}/(1 + \dot{\varepsilon}_{z0} t), \\ \dot{\varepsilon}_z = \partial V_z/\partial z = \dot{\varepsilon}_{z0}/(1 + \dot{\varepsilon}_{z0} t), \qquad \dot{\varepsilon}_{rz} = 0.5(\partial V_z/\partial r + \partial V_r/\partial z) = 0. \end{gathered} \tag{3}$$

The equality of the radial and tangential components of the strain-rate tensor and the absence of shear strains make it possible to obtain, using the Saint Venant–von Mises theory of plasticity, the following relations for the stress-tensor and stress-deviator components: $\sigma_r = \sigma_\theta$ and $S_{rz} = 0$. In this case, the equation of motion for the radial velocity component is reduced to the form $\rho_0 dV_r/dt = \partial\sigma_r/\partial r$ or, with allowance for relations (2), to the form $\partial\sigma_r/\partial r = (3/4)[\dot{\varepsilon}_{z0}/(1 + \dot{\varepsilon}_{z0} t)]^2 r$.

With allowance for the fact that the radial stress σ_r is zero at the free surface $r = R$ of the element, integration of the latter differential equation over the radial coordinate r at constant time t makes it possible to find the radial-stress distribution inside the element, whereas the von Mises plasticity condition, which is reduced in this case to the relation $\sigma_z - \sigma_r = Y_0$, yields a similar distribution of the axial stresses:

$$\sigma_r = -(3/8)\rho_0[\dot{\varepsilon}_{z0}/(1 + \dot{\varepsilon}_{z0} t)]^2 (R^2 - r^2); \tag{4}$$

$$\sigma_z = Y_0 - (3/8)\rho_0[\dot{\varepsilon}_{z0}/(1 + \dot{\varepsilon}_{z0} t)]^2 (R^2 - r^2). \tag{5}$$

Relations of the type of (4) and (5) are given in [5, 9].

Using expressions (1)–(5), we can derive energy relations for an incompressible inelastoplastic bar stretched under the conditions considered.

For example, the initial values of the kinetic energy of the radial W_{r0} and axial W_{z0} motion of the bar particles are found in accordance with the following expressions:

$$W_{z0} = \int_0^{l_0} 0.5\rho_0 \pi R_0^2 dz (\dot{\varepsilon}_{z0} z)^2 = m_b \dot{\varepsilon}_{z0}^2 l_0^2/6,$$

$$W_{r0} = \int_0^{l_0} 0.5\rho_0 2\pi r dr dz (-r\dot{\varepsilon}_{z0}/2)^2 = m_b \dot{\varepsilon}_{z0}^2 R_0^2/16,$$

where $m_b = \rho_0 \pi R_0^2 l_0$ is the mass of the bar. The total initial kinetic energy W_0 in the frame of reference related to the fixed end of the bar is expressed in the form

$$W_0 = W_{z0} + W_{r0} = m_b \dot{\varepsilon}_{z0}^2 l_0^2/6 + m_b \dot{\varepsilon}_{z0}^2 R_0^2/16. \tag{6}$$

Similarly, using the relation $\dot{\varepsilon}_z = \dot{\varepsilon}_{z0}/n$ for the current and initial axial-velocity gradient and also the relation $R = R_0/\sqrt{n}$ for the initial and current radii of the bar, we determine the total current kinetic energy W in the form

$$W = W_z + W_r = m_b\dot{\varepsilon}_{z0}^2 l_0^2/6 + m_b\dot{\varepsilon}_{z0}^2 R_0^2/(16n^3). \tag{7}$$

It follows from comparison of relations (6) and (7) that the kinetic energy of the axial motion of the bar particles remains invariable ($W_z = W_{z0}$) during stretching. However, the radial-motion kinetic energy W_r decreases rather abruptly in inverse proportion to the cube of the coefficient of current elongation n. The dynamic stretching of the inelastoplastic bar is accompanied by energy dissipation and by a mechanical-to-internal energy transition.

The magnitude of losses attributed to energy dissipation in plastic deformation is determined by integration of the energy equation in the adiabatic approximation $\dot{E} = \sigma^{ij}\dot{\varepsilon}_{ij}/\rho_0$, where E is the specific internal energy of the bar particles and σ^{ij} and $\dot{\varepsilon}_{ij}$ are the components of the stress- and strain-rate tensors, respectively. With relations (3)–(5) taken into account, the differential energy equation is reduced to the form $\dot{E} = (Y_0/\rho_0)[\dot{\varepsilon}_{z0}/(1 + \dot{\varepsilon}_{z0}t)]$ which allows us to find the current value of the internal energy for the whole bar:

$$E_d = m_b \int_0^t \dot{E}\, dt = m_b(Y_0/\rho_0)\ln n. \tag{8}$$

As follows from the character of the distribution of the axial stresses σ_z over the cross section of the bar, its stretching occurs in the presence of the axial force F_z which acts in each plane cross section of the bar including the end cross sections:

$$F_z = \int_0^R 2\pi r\, dr\sigma_z = \pi R^2[Y_0 - (3/16)\rho_0\dot{\varepsilon}_{z0}^2 R_0^2/n^3] = \pi R^2\sigma_{z\mathrm{av}}, \tag{9}$$

where $\sigma_{z\mathrm{av}}$ is the average axial stress over the bar cross section. The existence of the axial force F_z in the transverse cross section of the bar corresponds physically to the fact that the uniform stretching of an arbitrary isolated element of a SCJ takes place with a force interaction with the neighboring elements and is accompanied by the work done on them:

$$A_d = \int_0^t -F_z V_0\, dt.$$

The expression for this work has the following form:

$$A_d = -m_b(Y_0/\rho_0)\ln n - m_b\dot{\varepsilon}_{z0}^2 R_0^2(1 - n^3)/n^3. \tag{10}$$

It is easily seen that, for a dynamically stretching cylindrical inelastoplastic bar, the energy relations (6)–(8) and (10) correspond to the law of conservation of energy $W_0 = W + E_d + A_d$. Since the kinetic energy of the axial motion is conserved $W_z = W_{z0}$, upon stretching of the bar this law takes the form $W_{r0} = W_r + E_d + A_d$. In accordance with this, in the course of stretching the initial kinetic energy of the radial motion of the jet particles partly dissipates and becomes the internal thermal energy. In addition, during the process considered, this energy is partly spent to sustain the deformation conditions (the work done on the surrounding SCJ elements) and partly remains active as the current kinetic energy of the radial motion W_r.

Figure 4 shows some calculation results for the element of the middle section of a copper SCJ with initial radius $R_0 = 3.5$ mm, initial axial-velocity gradient $\dot{\varepsilon}_{z0} = 3.18 \cdot 10^5$ sec^{-1}, and yield point of the material $Y_0 = 2 \cdot 10^8$ Pa. The calculations were carried out under laboratory conditions to provide an idea of the quantitative aspects of the process of stretching of a SCJ element considered within the framework of the model of a cylindrical incompressible inelastoplastic bar.

Figure 4 shows distributions of the radial σ_r (curves 1 and 3) and axial σ_z (curves 2 and 4) stresses for times which correspond to the current coefficients of elongation $n_1 = 2.4$ and $n_2 = 3.5$ and also to the bar

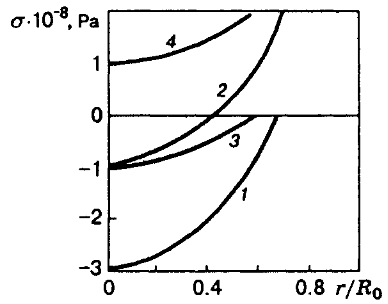

Fig. 4

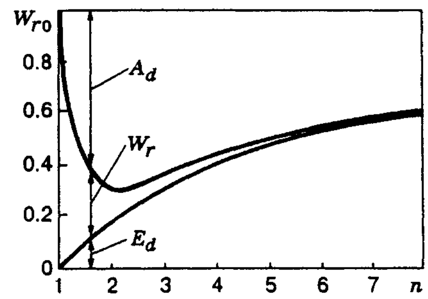

Fig. 5

radius $R_1 = 0.65R_0$ and $R_2 = 0.54R_0$. Clearly, in most parts of the cross section, the SCJ element stretches initially under conditions of all-around compression. The sole exception is the zone adjacent to the outer surface where the axial stresses are stretching stresses. In the near-axial cross-section zone, the compressing stresses can, however, be rather strong. They are maximal at the axis of the bar ($r = 0$) at the time of onset of the deformation process ($t = 0$) and are determined, according to (4) and (5), by the value of the complex $\rho_0\dot{\varepsilon}_{z0}^2 R_0^2$ which specifies the initial specific (per unit volume) kinetic energy of the radial motion. As the bar elongates, its stress state tends to a state of uniaxial extension, while the radial and tangential stresses decrease monotonically, remaining compressive at the same time [see formulas (4) and (5)]. It follows from (9) that under conditions of all-around compression, the bar is, on the average, stretched if the initial specific kinetic energy of the radial motion exceeds the yield point: $\rho_0\dot{\varepsilon}_{z0}^2 R_0^2/Y_0 \geqslant 16/3$.

Figure 5 shows the energy balance for the same SCJ element as in Fig. 4. One can see that the radial-motion kinetic energy W_r decreases rather abruptly and becomes negligible in comparison with the internal energy E_d. A considerable portion of the kinetic energy is spent for the work A_d done by this element on the neighboring elements. In this case, A_d increases at the initial stage of stretching — the element deforms and does work under conditions of all-around compression, as if it "pushes" the neighbors. However, beginning with a certain value of the coefficient of elongation, the work A_d decreases — the SCJ element is stretched by the stretching axial force F_z of interaction with the neighboring elements.

The above results, which were obtained within the model of an incompressible inelastoplastic bar, have made it possible to find the range of the most important stretching and destruction characteristic of plastically destructible SCJ, namely, the coefficient of ultimate elongation:

$$\sqrt[3]{(3/16)\rho_0\dot{\varepsilon}_{z0}^2 R_0^2/Y_0} < n_{\text{ult}} < \exp\left[\rho_0\dot{\varepsilon}_{z0}^2 R_0^2/(16Y_0)\right].$$

Here the lower bound of the range is obtained from the condition of transition of a stress state upon stretching caused by the all-around compression, and the upper bound is obtained from the condition of complete dissipation of the initial kinetic energy of the radial motion under the assumptions that $A_d = 0$ and $W_r/W_{r0} \ll 1$. For the current coefficients of elongation n smaller than the lower bound at which the material inside the jet is compressed all-around, one should apparently exclude the possibility of developing and moreover completing the process of necking. The coefficients of elongation that are larger than the upper bound cannot be realized because of the "veto" imposed by the law of conservation of energy. The latter statement can be substantiated as follows.

Experimental data show that in plastic failure, high-gradient SCJ break up into several tens of individual nongradient elements. In this case, the initial length l_0 of each SCJ section formed during failure is, as a rule, of the order of tenths of the initial radius R_0 for the corresponding section. Therefore, at the moment of onset of its deformation, the "surplus" kinetic energy of such a section (the kinetic energy of the axial and radial motion of material in the frame of reference related to the center of masses of this SCJ section and, later, the kinetic energy of the individual nongradient element formed from this section) is mainly determined by the kinetic energy of the radial motion, whereas its axial portion is negligible [see relation (6)].

A separate nongradient element that is formed in plastic failure has no "redundant" kinetic energy — each of these elements moves in space as a rigid unit. Therefore, for the SCJ which initially represents a set of interrelated sections and, eventually, is a set of separate elements formed from these sections, it is correct to suggest that the initial kinetic energy W_{r0} of the radial motion serves as an upper bound for the energy spent for deformation of the corresponding SCJ elements and determines the maximum theoretically possible value of the coefficient of ultimate elongation n_{ult}. Meanwhile the work A_d done by the interacting sections on one another should not have an effect on the maximum possible ultimate elongation and can be set equal to zero in an estimation, because the interacting forces between the sections which form individual elements are internal forces with respect to the SCJ as a whole and have no effect on its energy balance.

Thus, in the present paper, we have analyzed a physicomathematical model for uniform stretching of a SCJ element as an incompressible cylindrical inelastic-perfectly-plastic bar. The model has made it possible to get some ideas on the character of variation of the kinematic, dynamic, and energetic characteristics of this process and also to establish bounds for finding the basic quantitative characteristic of jet fracture, namely, the coefficient of ultimate elongation. The next step to refine these ideas should apparently be connected with taking account, in the model, of the compressibility and elastoplastic characteristics inherent in the material of shaped-charge jets.

REFERENCES

1. M. A. Lavrent'ev, "Shaped charge and principles of its operation," *Usp. Mat. Nauk*, **12**, No. 4, 41–56 (1957).
2. K. P. Stanukovich (ed.), *Physics of Explosion* [in Russian], Nauka, Moscow (1975).
3. P. C. Chou and J. Carleone, "The stability of shaped-charge jets," *J. Appl. Phys.*, **48**, No. 10, 4187–4194 (1977).
4. E. A. Karabut, "The coefficient of elongation of shaped-charge jets," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 6, 167–169 (1983).
5. J. M. Walsh, "Plastic instability and particulation in stretching metal jets," *J. Appl. Phys.*, **56**, No. 7, 1997–2006 (1984).
6. J. P. Curtis, "Axisymmetric instability model for shaped-charge jets," *J. Appl. Phys.*, **61**, No. 11, 4978–4985 (1987).
7. D. C. Pack, "On the perturbation and breakup of a high-speed elongating metal jet," *J. Appl. Phys.*, **62**, No. 6, 1864–1871 (1988).
8. L. A. Romero, "The instability of rapidly stretching plastic jets," *J. Appl. Phys.*, **65**, No. 8, 3006–3016 (1989).
9. R. R. Karpp, L. M. Hull, and M. L. Price, "A technique for estimating the strength of materials in stretching shaped-charge jets," in: 15th Int. Symp. on Ballistics, Jerusalem, Israel, May 21–24 (1995), pp. 175–182.